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We have investigated the efficiency of pulsed Raman sideband cooling in the presence of multiple decay and excitation channels. By applying sum rules we identify parameter regimes in which multiple scattering of photons can be described by an effective wave vector. Using this method we determine the rate of heating caused by optical pumping inside and outside the Lamb-Dicke regime. On this basis we discuss also the efficiency of a recently proposed scheme for ground-state cooling outside the Lamb-Dicke regime [G. Morigi, J.I. Cirac, M. Lewenstein, and P. Zoller, *Europhys. Lett.* **39**, 13 (1997)].

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I. INTRODUCTION

Laser-cooling [1] allows to cool ions and atoms to very low temperatures. For this purpose, the full knowledge of the effects of the various physical parameters determining the cooling process is very important. Among the various schemes, Raman sideband cooling has been demonstrated to be a very successful technique for preparing atoms in the ground state of a harmonic potential [2]. This cooling method exploits two stable or metastable atomic internal levels, which we call $|g\rangle$ and $|e\rangle$, connected by dipole transitions to a common excited state $|r\rangle$. The transitions are usually driven by alternating pulses. A typical sequence alternates a coherent pulse, in which the atom is coherently transferred from $|g\rangle$ to $|e\rangle$ via a properly designed Raman pulse, with a re-pumping pulse, in which the atom is incoherently re-scattered to $|g\rangle$ by means of a laser resonant with $|e\rangle \rightarrow |r\rangle$. A change of the motional state during the repumping is a process of higher order in the ratio ω_R/ν of the recoil frequency $\omega_R = \hbar k^2/2m$ and the trap frequency ν , with m being the mass of the atom and k the wave vector of the one-photon transition. In the Lamb-Dicke regime, where $\omega_R \ll \nu$, the probability for a change of the motional state is negligible and therefore, on the average, the system is cooled at a rate of one phonon of energy $\hbar\nu$ per cooling cycle. Since there is a finite probability for the atom to be returned to the state $|e\rangle$ instead of being repumped, a number of incoherent scattering events may be required before the atom is finally scattered into $|g\rangle$, which significantly increases the motional energy at the end of the optical pumping, reducing the cooling efficiency. Furthermore, since two and three level schemes are realized using Zeeman or hyperfine substates, decays from $|r\rangle$ into other electronic substates can occur, leading to additional heating.

In this work we quantify the effect of a finite branching ratio in pulsed Raman sideband cooling by calculating the average shift and diffusion of the vibrational energy distribution at the end of an incoherent pumping pulse. It should be pointed out that theoretical studies on laser-cooling for multilevel ions exist, which systematically include the branching ratio in their treatments [3–5]. Those studies have focussed on the Lamb-Dicke regime and on certain cooling schemes. Here, we single out the effect of the branching ratio on cooling for an arbitrary ratio ω_R/ν by applying sum rules. Hence, we infer the cooling efficiency in the Lamb-Dicke regime and we discuss the result outside the Lamb-Dicke regime in connection with the proposal in [6]. In particular, we show that in some parameter ranges the average effect of the multiple photon scattering can be described with an effective wave vector k_{eff} for the “effective” two-level transition $|e\rangle \rightarrow |g\rangle$ [3].

This article is organized as follows. In Section 2 we introduce the model for the evolution of a trapped ion during the repumping pulse in a Raman transition, and we evaluate the average shift and variance of the ion energy at the end of the pulse. In Section 3 we extend our analysis to cases where the channels of decay are multiple. In Section 4 we draw some conclusions, and in the Appendix we report the details of our calculations.

II. MODEL

We consider a three level atom as in Fig. 1, whose internal levels are a ground state $|g\rangle$, stable or metastable state $|e\rangle$ and excited state $|r\rangle$ of radiative width γ ; $|g\rangle \rightarrow |r\rangle$, $|e\rangle \rightarrow |r\rangle$ are dipole transitions, with respective probabilities of decay p_g , p_e , where $p_g + p_e = 1$. A laser resonantly drives the transition $|e\rangle \rightarrow |r\rangle$ with Rabi frequency Ω_e . In the following we assume the wave vectors for both transitions to be equal to k , which is a good approximation if, e.g., $|e\rangle$

and $|g\rangle$ are hyperfine components of the ground state. We study the ion motion in one-dimension. The master equation for the atomic density matrix ρ_3 is written as ($\hbar = 1$):

$$\frac{d}{dt}\rho_3 = -i[H_0 + V, \rho_3] + L\rho_3, \quad (1)$$

where H_0 has the form:

$$H_0 = \delta|e\rangle\langle e| + \nu a^\dagger a. \quad (2)$$

Here, δ is the detuning of the laser on the $|e\rangle \rightarrow |r\rangle$ transition, which we take to be zero, and ν is the frequency of the harmonic oscillator which traps the ion along the x -direction, with a, a^\dagger annihilation and creation operator, respectively. The interaction of the ion with the laser light is described in the dipole approximation by the operator V :

$$V = \frac{\Omega_e}{2} (A_e^+ e^{ikx} + \text{h.c.}), \quad (3)$$

with $A_j^+ = |r\rangle\langle j|$ (with $j = e, g$) dipole raising operator, A_j^- its adjoint, and x the position of the atom. In writing (3), (1) we have applied the Rotating Wave Approximation and we have moved to the inertial frame rotating at the laser frequency. Finally, the relaxation super-operator has the form

$$L\rho_3 = -\frac{\gamma}{2} (|r\rangle\langle r|\rho_3 + \rho_3|r\rangle\langle r|) + \sum_{j=g,e} p_j \gamma \int_{-1}^1 du N(u) A_j^- e^{-iku} \rho_3 e^{iku} A_j^+,$$

where $N(u)$ is the dipole pattern of the spontaneous emission, which we take $N(u) = 3/8(1+u^2)$. In the limit $\Omega_e \ll \gamma$ we can eliminate the excited state $|r\rangle$ in second order perturbation theory [7], and reduce the three-level scheme to a two level one, with excited state $|e\rangle$ and linewidth $\gamma_e = \Omega_e^2/\gamma$ [3]. In the limit $\gamma \gg \nu$ the master equation for the density matrix ρ , projection of ρ_3 on the subspace $\{|e\rangle, |g\rangle\}$, can be rewritten as [8]:

$$\frac{d}{dt}\rho = -i[H_{\text{eff}}\rho - \rho H_{\text{eff}}^\dagger] + \gamma_e [J_e\rho + J_g\rho], \quad (4)$$

with H_{eff} effective Hamiltonian

$$H_{\text{eff}} = H_0 - i\frac{\gamma_e}{2}|e\rangle\langle e|, \quad (5)$$

and with $J_e\rho, J_g\rho$ jump operators, defined as:

$$J_j\rho = p_j (\sigma_{je} [J'\rho] \sigma_{ej}) \quad \text{with} \quad j = g, e, \quad (6)$$

where $\sigma_{ij} = |i\rangle\langle j|$ and where

$$J'\rho = \int_{-1}^1 du N(u) e^{-ik(1+u)x} \rho e^{ik(1+u)x}. \quad (7)$$

The solution of Eq. (4) can be written as follows [8]:

$$\begin{aligned} \rho(t) = & S(t)\rho(0) + \gamma_e \int_0^t dt_1 S(t-t_1) J S(t_1) \rho(0) + \dots \\ & + \gamma_e^n \int_0^t dt_1 \int_0^{t_1} dt_2 \dots \int_0^{t_{n-1}} dt_n S(t-t_1) J S(t_1-t_2) J_e \dots J_e S(t_n) \rho(0) + \dots, \end{aligned} \quad (8)$$

with $J = J_e + J_g$, and $S(t)$ is the propagator for the effective Hamiltonian:

$$S(t)\rho(0) = e^{-iH_{\text{eff}}t} \rho(0) e^{iH_{\text{eff}}^\dagger t}. \quad (9)$$

In Eq. (8) the successive contributions to the multiple scattering event are singled out: The first term on the RHS corresponds to the case in which at time t no spontaneous decay has occurred. The second term describes a single

scattering event, and the n -th term $n - 1$ scattering events. The trace of each term corresponds to the probability associated with each event, and we can thus interpret Eq. (8) as the sum over all the possible paths of the scattering event weighted by their respective probabilities. At $t \rightarrow \infty$, $\rho(t) \rightarrow \rho_S$, the atom is in $|g\rangle$ and $\langle e|\rho_S|e\rangle = 0$. For a pulse of duration $t \gg 1/\gamma_e$ we can replace t by ∞ in the integrals of Eq. (8) and assume that the atom has been scattered into $|g\rangle$ at the end of the pulse. Now, each term on the RHS of Eq. (8) corresponds to the path associated with a certain number of scattering events into $|e\rangle$ before the atom is finally scattered into $|g\rangle$. Through (8) we can evaluate the shift and the variance of the energy distribution at the end of the repumping pulse, which are defined as:

$$\Delta E = \text{Tr}\{(H_{\text{mec}} - E_0)\rho_S\}, \quad (10)$$

$$\sigma_E = \sqrt{\text{Tr}\{(H_{\text{mec}} - E_0 - \Delta E)^2 \rho_S\}}, \quad (11)$$

where $H_{\text{mec}} = \nu a^\dagger a$ and E_0 is the initial motional energy of the atom.

A. Evaluation of the average shift and diffusion

For simplifying the form of the discussion presented below, we rewrite the operator J' as follows:

$$J'\rho = \tilde{J}\rho + \hat{J}\rho, \quad (12)$$

where \tilde{J} , \hat{J} are defined as:

$$\tilde{J}\rho = \sum_l |l\rangle\langle l| [J\rho] |l\rangle\langle l|, \quad (13)$$

$$\hat{J}\rho = \sum_l \sum_{l_1, l_1 \neq l} |l\rangle\langle l| [J\rho] |l_1\rangle\langle l_1|, \quad (14)$$

and where $\{|l\rangle\}$ is the basis of eigenstates of the harmonic oscillator. For $\rho(0) = |e\rangle\langle e| \otimes \mu(0)$, with $\mu(0)$ initial distribution over the motional states, and according to Eq. (8) the steady state distribution has the form:

$$\mu_\infty = \sum_{m=1}^{\infty} p_g p_e^{m-1} \tilde{J}^m \mu(0) + F(\tilde{J}, \hat{J})\mu(0), \quad (15)$$

where $\mu_\infty = \langle g|\rho_S|g\rangle$ is the final distribution over the motional states. The first term in the RHS of (15) is the sum over all paths from $|e\rangle$ into $|g\rangle$, where after each jump the density operator is diagonal in the basis $\{|l\rangle\}$, whereas the second term contains all other paths. These latter terms can be neglected [9], and for $\mu(0) = |n\rangle\langle n|$ the following relation holds:

$$\langle s|\mu_\infty|s\rangle \approx \langle s|\left[\sum_{m=1}^{\infty} \tilde{J}^m |n\rangle\langle n|\right]|s\rangle = D_n(s). \quad (16)$$

Here, $D_n(s)$ is the probability for the atom to be found in the state $|g, s\rangle$ at $t \rightarrow \infty$, given the initial state $|e, n\rangle$ at $t = 0$. Using the explicit form (13) of \tilde{J} in (16), $D_n(s)$ has the form:

$$D_n(s) = p_g \sum_{m=0}^{\infty} p_e^{m-1} \sum_{k=0}^{\infty} \cdot \int_{-1}^1 du_1 \dots \int_{-1}^1 du_m N(u_1) \dots N(u_m) \quad (17)$$

$$\cdot \sum_{k_1=0}^{\infty} \dots \sum_{k_{m-1}=0}^{\infty} |\langle s|e^{ika_0(1+u_1)(a^\dagger+a)}|k_1\rangle|^2 \dots |\langle k_{m-1}|e^{ika_0(1+u_m)(a^\dagger+a)}|n\rangle|^2,$$

where we have used the relation $x = a_0(a^\dagger + a)$, with $a_0 = \sqrt{1/2m\nu}$ size of the ground state of the harmonic oscillator. Substituting (17) into Eqs. (10), (11), and applying the commutation properties of a, a^\dagger [see the Appendix], we find:

$$\langle \Delta E \rangle = \nu \eta^2 \frac{7}{5} \frac{1}{1 - p_e}, \quad (18)$$

$$\sigma_E^2 = \nu^2 \left[\eta^2 \frac{7}{5} (2n + 1) \frac{1}{1 - p_e} + \left(\frac{7\eta^2}{5} \right)^2 \frac{58}{49} \frac{p_e}{(1 - p_e)^2} \right], \quad (19)$$

where $\eta = ka_0 = \sqrt{\omega_R/\nu}$ is the Lamb-Dicke parameter.

Equation (18) represents the average shift to the vibrational energy at the end of the repumping pulse. For $p_e = 0$ it corresponds to the average recoil energy ω'_R associated with one incoherent Raman scattering into $|g\rangle$. In this case, the second term in the RHS of Eq. (19) vanishes, and Eqs. (18), (19) describe the scattering of one photon of wave vector $k' = \sqrt{7/5}k$ on the effective two-level transition $|e\rangle \rightarrow |g\rangle$. Similarly for $p_e > 0$ an effective wave vector k_{eff} can be defined for the incoherent scattering on the two-level transition $|e\rangle \rightarrow |g\rangle$, which has the form

$$k_{\text{eff}} = \frac{k'}{\sqrt{1-p_e}} = \sqrt{\frac{7}{5}} \frac{k}{\sqrt{1-p_e}}. \quad (20)$$

Thus, k_{eff} describes the average mechanical effect on the ion resulting from the multiple scattering of photons during the repumping pulse in a Raman transition with branching ratio $(1-p_e)/p_e$: This description is valid in the limit in which we may neglect the second term in the RHS of (19), *i.e.* for p_e and/or η sufficiently small. In Fig. 2 the first term of RHS of Eq. (19) is compared with the complete expression for $n = 0$, for different values of the Lamb-Dicke parameter and as a function of p_e . Here, we see that k_{eff} characterizes the scattering process for almost any branching ratio in the Lamb-Dicke regime, whereas for $\eta = 0.6$ an appreciable difference is already visible at $p_e = 0.2$.

From (20) we can define the effective Lamb-Dicke parameter $\eta_{\text{eff}} = k_{\text{eff}}a_0$ describing an incoherent scattering into the state $|g\rangle$. This parameter provides an immediate estimate of the effect of the branching ratio on cooling. For $\eta\sqrt{n} \ll 1$, if $\eta_{\text{eff}}\sqrt{n} \ll 1$ the system is still in the Lamb-Dicke regime once it has been finally scattered into $|g\rangle$. Furthermore, the coarse-grained dynamics of the system can be described by a rate equation for the motional states $|n\rangle$ projected onto $|g\rangle$, where the rate of cooling (heating) is the real part of the sum of two terms: one corresponding to the component of the fluctuation spectrum of the dipole force at frequency ν ($-\nu$), the other to the diffusion coefficient due to spontaneous emission from the excited state [3,4]. This latter term is proportional to the squared Lamb-Dicke parameter for the incoherent scattering, and thus in our case to η_{eff}^2 . From the well-known solution of the rate equation [10], the diffusion term affects the steady state average vibrational number $\langle n \rangle$, which is proportional to the diffusion coefficient.

Outside the Lamb-Dicke regime, when ω_R is comparable to, or larger than, ν , there are no established ground-state laser-cooling techniques for trapped atoms. Here, we discuss our result in connection to the proposal in [6]. There, a cooling scheme similar to Raman sideband cooling has been presented, where pulses which pump the atoms to the ground state alternate with pulses confining the atoms to a limited region of motional energy. These confinement pulses have two-photon detuning δ_c to the red of the two-photon resonance frequency, where $\delta_c \approx \omega'_R$. Then, the presence of a branching ratio must be taken into account by choosing $\delta_c \approx \Delta E$. In this regime, pulses which efficiently counteract the average kick ΔE can be designed, provided that the following condition is fulfilled:

$$k_x^{\text{coh}} \geq k_{\text{eff}}, \quad (21)$$

where k_x^{coh} is the projection on x of the two-photon wave vector of the coherent pulse. For two counterpropagating beams parallel to x , $k_x^{\text{coh}} = 2|k|$ and (21) is fulfilled for $p_e \leq 13/20$, *i.e.* up to branching ratios $p_e/p_g \approx 2$. Finally, outside the Lamb-Dicke regime the second term in the RHS of Eq. (19) cannot be neglected. Hence, the diffusion is larger, and the efficiency of cooling may decrease dramatically as p_e increases.

III. EXTENSION TO MULTI-LEVEL SCHEMES

In the following, we show that the average heating associated with the repumping pulse in multilevel-schemes can be described in the same way as discussed in the previous sections.

Let us consider the level-scheme of Fig. 3(a), where we have added to the scheme of Fig. 1 a further channel of decay from $|r\rangle$ into the stable or metastable state $|1\rangle$, with probability of decay p_1 such that $p_1 + p'_e + p'_g = 1$, where p'_e , p'_g are the probability of decay onto $|e\rangle$, $|g\rangle$, respectively. A laser resonantly drives the transition $|1\rangle \rightarrow |r\rangle$ with Rabi frequency Ω_1 . For $\Omega_e, \Omega_1 \ll \gamma$ the state $|r\rangle$ can be adiabatically eliminated from the equations of motion. In this limit the Master Equation acquires the form

$$\begin{aligned} \frac{d}{dt}\rho = & -i [H_{\text{eff}}\rho - \rho H_{\text{eff}}^+] \\ & + p'_e \gamma' J_e \rho + p_1 \gamma' J_1 \rho + p'_g \gamma' J_g \rho, \end{aligned} \quad (22)$$

where $\gamma' = \gamma_e + \gamma_1$, with $\gamma_j = \Omega_j^2/\gamma$. The effective Hamiltonian is now:

$$H_{\text{eff}} = H_0 - i\frac{\gamma_e}{2}|e\rangle\langle e| - i\frac{\gamma_1}{2}|1\rangle\langle 1|, \quad (23)$$

and the jump operators have the form:

$$J_i \rho = p_i \sum_{j=1,e} \sigma_{ij} \left(\tilde{J} + \hat{J} \right) \sigma_{ji}, \quad (24)$$

with $i = 1, e, g$. The solution at $t \rightarrow \infty$ can be written as:

$$\mu_\infty = \sum_{m=1}^{\infty} p'_g (p'_e + p_1)^{m-1} \tilde{J}^m \mu(0). \quad (25)$$

Hence, the shift and variance have the form evaluated in Eqs. (18), (19) where now the probability p_g, p_e are defined as $p_e = p'_e + p_1$, $p_g = p'_g$ ($p_g + p_e = 1$). In a similar way we have evaluated these quantities for schemes like the one shown in fig. 3(b), where a second excited state $|2\rangle$ is coupled to $|e\rangle$ via the same recycling laser tuned on the transition $|1\rangle \rightarrow |r\rangle$. For simplifying the treatment, we assume that a fourth laser resonantly drives the transition $|1\rangle \rightarrow |2\rangle$ with Rabi frequency Ω (grey arrow in fig. 3(b)). Thus, for low saturation Eq. (22) describes the dynamics, where now $\gamma_i = \gamma_i^{(r)} + \gamma_i^{(2)}$ ($i = e, 1$), with $\gamma_i^{(j)}$ being the rate of scattering through the excited state $|j\rangle$ ($j = r, 2$). Assuming that Ω is such that $\gamma_e^{(r)}/\gamma_e^{(2)} = \gamma_1^{(r)}/\gamma_1^{(2)} = a$, the solution in Eqs. (18), (19) applies to this case too, where now p_e is defined as:

$$p_e = (p'_e + p_1) \frac{a}{1+a} + \frac{1}{1+a}, \quad (26)$$

and the probability p_g of decaying into $|g\rangle$ is $p_g = 1 - p_e$.

The result (26) shows that the total heating is minimum for $a \gg 1$, which can be obtained by choosing properly the laser intensity of the repumping lasers, or simply by removing degeneracies in the Zeeman multiplet, for example with the help of a magnetic field.

IV. CONCLUSIONS

We have studied the motional heating associated with a finite branching ratio and in the presence of multiple decay and excitation channels at the end of a repumping pulse in Raman sideband cooling. The first and second moments of the final energy distribution has been evaluated analytically, and the effect of the branching ratio has been singled out. We have shown that in a certain range of parameters the diffusion can be described with an effective wave vector k_{eff} , corresponding to an effective Lamb-Dicke parameter η_{eff} for the incoherent scattering on the two-level transition $|e\rangle \rightarrow |g\rangle$. Finally, on the basis of this result we have discussed the efficiency of Raman sideband cooling and of a recent proposal of ground-state cooling outside the Lamb-Dicke regime [6].

Analogous sum rules and considerations can be applied to Raman cooling for free atoms [11]. In that case the calculations are much simpler, since the total momentum of radiation and atom is a conserved quantity in the scattering event.

In general, these results can be applied to cooling schemes in multilevel atoms.

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VI. APPENDIX

Using (16), we rewrite (10) and (11) as:

$$\langle \Delta E \rangle = \nu \sum_{m=0}^{\infty} B_m^n, \quad (27)$$

$$\sigma_E^2 = \nu^2 \sum_{m=0}^{\infty} A_m^n, \quad (28)$$

where we have introduced the quantities

$$A_m^n = \sum_{s=0}^{\infty} (s - n - \Delta E/\nu)^2 \langle s | \left[\tilde{J}^m |n\rangle \langle n| \right] |s\rangle, \quad (29)$$

$$B_m^n = \sum_{s=0}^{\infty} (s - n) \langle s | \left[\tilde{J}^m |n\rangle \langle n| \right] |s\rangle. \quad (30)$$

Using Eq. (17), Eq. (30) is rewritten as:

$$B_m^n = p_g p_e^{m-1} \sum_{k_1=0}^{\infty} (k_1 - n) \int_{-1}^1 du_1 \dots \int_{-1}^1 du_m N(u_1) \dots N(u_m) \\ \cdot \sum_{k_2=0}^{\infty} \dots \sum_{k_m=0}^{\infty} |\langle k_1 | e^{i\eta(1+u_1)(a^\dagger+a)} |k_2\rangle|^2 \dots |\langle k_m | e^{i\eta(1+u_m)(a^\dagger+a)} |n\rangle|^2. \quad (31)$$

The sum over k_1 can be contracted by observing that $k_1 |k_1\rangle \langle k_1| = a^\dagger a |k_1\rangle \langle k_1|$. Then, using the commutation properties of the bosonic operators a, a^\dagger and the closure relation for the eigenstates of the harmonic oscillator, Eq. (31) takes the form:

$$B_m^n = p_g p_e^{m-1} \int_{-1}^1 du_1 \dots \int_{-1}^1 du_m N(u_1) \dots N(u_m) \\ (-n + \eta^2(1+u_1)^2 + \sum_{k_2=0}^{\infty} \dots \sum_{k_{m-1}=0}^{\infty} k_2 |\langle k_2 | e^{i\eta(1+u_2)(a^\dagger+a)} |k_3\rangle|^2 \dots |\langle k_{m-1} | e^{i\eta(1+u_m)(a^\dagger+a)} |n\rangle|^2). \quad (32)$$

Repeating then the procedure shown in Eqs. (31),(32) for each index k_i , we finally obtain:

$$B_m^n = p_g p_e^{m-1} \frac{7}{5} \eta^2 m. \quad (33)$$

Analogously, A_m^n has the form:

$$A_m^n = p_g p_e^{m-1} \left(\frac{7}{5} \eta^2 (2n+1)m + \left(\eta^2 \frac{7}{5} \right)^2 \frac{29}{49} m(m-1) \right). \quad (34)$$

Substituting now (33), (34) into Eqs. (27), (28) and summing over m we finally obtain Eqs. (18), (19).

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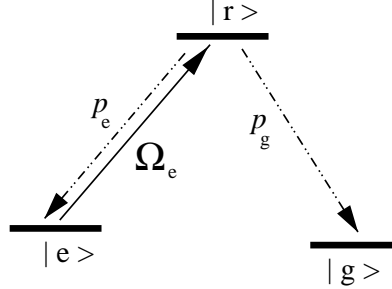


FIG. 1. Level scheme.

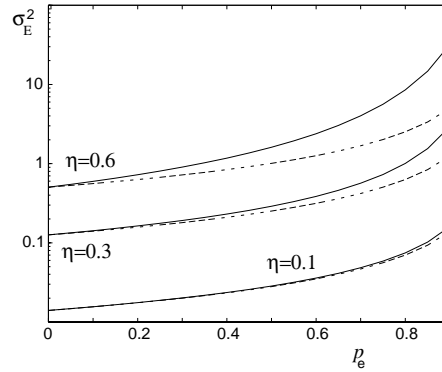


FIG. 2. Plot of σ_E^2 (solid line), and of the first term on the RHS of Eq. (19) (dashed line) as a function of p_e for Lamb-Dicke parameter $\eta = 0.1, 0.3, 0.6$ and for $n = 0$.

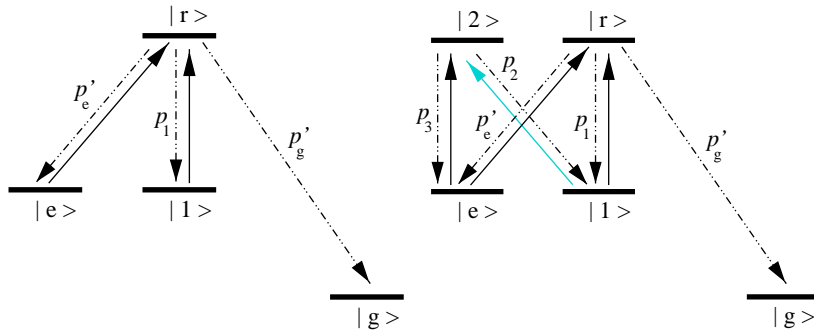


FIG. 3. (a) Level scheme with $|g\rangle, |e\rangle, |1\rangle$, stable or metastable states, $|r\rangle$ excited state of radiative width γ and probability of decaying in the three ground states p'_g, p'_e and p_1 , respectively. Two lasers couple $|e\rangle$ and $|1\rangle$ to $|r\rangle$; (b) Level scheme as in (a) with the addition of the excited state $|2\rangle$ with decay probability on $|1\rangle, |e\rangle$ equal to p_2, p_3 , respectively, $p_2 + p_3 = 1$. Two lasers couple $|e\rangle$ and $|1\rangle$ to $|2\rangle$.